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Financial Weather Options for Crop Production

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Abstract

Weather derivatives based on heating degree days or cooling degree days have been traded in financial markets for more than 10 years. Although used by the energy sector, agricultural producers have been slow to adopt this technology even though agriculture is particularly vulnerable to weather uncertainty. In agriculture, few studies have focused on the pricing of weather derivatives for hedging weather risks for crop production. In this study, we employ data from an earlier study of climate on corn yields in northern China to compare different methods for pricing weather options based on growing degree days (GDDs). For pricing weather options, we investigate the use of weather indexes based on an econometric approach, a mean reverting stochastic process, and simple historical averages (burn analysis). For the econometric model, we use a sine function to estimate expected GDDs. The stochastic model is also based on the sine function, but employs Monte Carlo simulation with mean-reversion parameters to predict daily average temperatures; the reversion parameters are estimated using three alternative methods. For the historical approach, a 10-year moving average of GDDs is used. Results for the period 2001-2011 indicate that the historical average method fits actual GDDs best, followed in order by the stochastic process with a high mean reversion speed (0.9763), the econometrically estimated sine function, and the stochastic processes with medium (0.2698) and low (0.02399) mean reversion speeds. Depending on the method used, premiums for weather derivative options vary from $21.27 to $24.39 per standard deviation in GDD.

Key Words: Stochastic process for pricing weather options; growing degree days; agricultural finance

JEL Categories: Q14, G11, G12, G32

1. INTRODUCTION

Financial weather derivatives and weather-indexed insurance are alternative private-sector instruments that can be used to hedge production risks related to weather outcomes. Payoffs depend on a weather index that has been carefully chosen to represent the weather conditions against which protection is being sought. The problems of moral hazard and adverse selection that exist in traditional crop insurance disappear since the value of the weather index does not depend on the individual actions of market participants. Although the two hedging methods – weather derivatives and weather indexed insurance – are essentially similar, there exist mature exchange markets for some financial weather derivatives while weather-indexed insurance relies solely on over-the-counter (OTC) contracts. Another important difference is that financial weather derivatives not only provide economic agents impacted by weather (e.g., farmers, energy firms) with a tool for hedging weather risks, but also provide an investment instrument that participants in financial markets can purchase for diversifying their portfolios.

Trading in financial weather derivatives began in 1997, with an OTC contract based on heating degree days (HDDs) struck between Koch Industrial and Enron Corporation (Brockett et al. 2007). Since then, trading has grown rapidly as the Chicago Mercantile Exchange (CME) began offering financial exchange-traded weather derivatives based on two weather indexes, HDDs and CDDs (cooling degree days) (Considine 2009). A party wishing to hedge against adverse weather can purchase an option on one of these two weather indexes: A call option can be claimed when the value of the weather index is above a specified exercise or strike value, while a put option can be claimed when the value of the weather index is below a specified value. The cost of acquiring an option is its premium. For call or put options, buyers take a long position, while sellers take a short position.

Weather derivatives can be used to protect against crop losses associated with cold weather, extreme heat, and/or too much or too little precipitation, although financial rainfall products are generally traded OTC. For example, a crop producer could insure against too little growing season warmth by holding a put option based on growing degree days (GDDs), which measure the dependence of crops on warmth and are defined with respect to a 5^{o}C or 10^{o}C threshold. Alternatively, if precipitation is a concern, an option on cumulative rainfall (CR) can be purchased. A farmer could hedge against too few GDDs or too little CR by purchasing a put option that reduces the financial risk of low crop yield. If the realized weather outcome is at or above the strike value, the farmer would not exercise the option and lose the premium paid for the option contract; in that case, yields are likely higher than expected, which would more than compensate for the premium. If the weather outcome is below the strike value, the farmer receives a payout to compensate for the lower yields and reduced revenue from the adverse weather.

In this paper, we examine potential pricing of weather derivatives in China, which is the second largest maize producing country in the world after the United States (FAO 2010). Crop yields in northern China (mainly areas in Inner Mongolia and Shaanxi province) are highly dependent on growing season weather conditions, especially heat conditions during the growing season (Sun and van Kooten 2013). Therefore, farmers could use a GDD-based financial weather product to mitigate weather risk.

A number of studies have focused on methods for pricing weather derivative contracts, including Alaton et al. (2002), Brody et al. (2002), Campbell and Diebold

(2005), and Jewson et al. (2005). In these studies, burn analysis and parametric or non-parametric methods were used to specify a probability distribution of the weather index, or, alternatively, a stochastic process was employed to model weather outcomes. Not surprisingly, most studies of weather derivatives focused on market-based HDD or CDD indexes in the energy sector (Huang et al. 2008；Goncu 2011; Schiller et al. 2012). In agriculture, where financial weather derivatives have not been adopted on the same scale as in the energy sector, studies have looked at rainfall or heat index-based weather derivatives, using historical data to construct such indexes (Turvey 2001; Stoppa and Hess 2003; Vedenov and Barnett 2004；Musshoff et al. 2011; Sun and Lou 2013).

The main objective of the current study is to examine three pricing methods for weather derivatives and compare them on the basis of historic weather conditions and weather predictions. The methods we employ to price weather derivatives based on GDDs are a weather index distribution method using historic averages (burn analysis), an estimated non-stochastic sine function, and a stochastic process with Monte Carlo simulation (and three approaches for estimating the mean-reverting parameter). Our application is to a major corn growing region in northern China, using historic weather data to estimate the required relationships; to do so, we rely on information from an earlier study on weather effects on corn yields in northern China (Sun and van Kooten 2013).

The study is structured as follows. We begin in the next section with a discussion of the development of daily average temperatures, followed by the stochastic method for simulating daily average temperatures and description of a weather index distribution method to price weather derivatives. We end by discussing and analyzing the results, and making some concluding remarks.

2. DATA DESCRIPTION

Weather data are from the China Meteorological Data Sharing System. A plot of daily average temperatures for the period 2001 to 2011 at Etuokeqi in the Inner Mongolia Autonomous Region is provided in Figure 1. This 11-year period includes two leap years and has 4,017 observations; the daily average temperature over this period is 8.0 ^{0}C, with a standard deviation of 11.99 ^{0}C. The minimum and maximum temperatures are -22.4 ^{0}C and +29.9 ^{0}C, respectively, while daily average temperatures range from -15 ^{o}C in winter to 25 ^{o}C in summer. The figure illustrates the seasonality in average daily temperature movements, indicating in particular its similarity to a sine function.

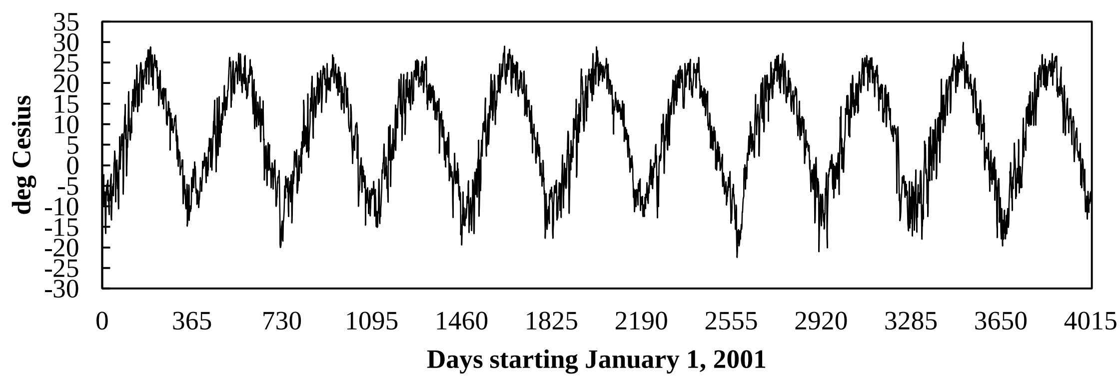

Figure 1: Daily Average Temperatures, 2001 through 2011, Etuokeqi, Inner Mongolia (107º59´E, 39º6´N)

Growing degree days are a measure of the heat to which crops are exposed during the growing season. In an earlier study, Sun and van Kooten (2013) show that corn yields are negatively impacted when growing season GDDs are too low or high, with GDD defined as: GDD = $\sum_{d=1}^{D} \text{Max}(0, T_d - 10)$, where D (=153) refers to the number of days in the growing season (May to September) and T_d is the average temperature on day d. For the 11 years in our sample, the average growing-season GDDs is 1,449.78 ^{0}C with a standard deviation of 78.97 ^{0}C, and minimum and maximum values of 1,294.1 ^{0}C and 1,584.4 ^{0}C, respectively. A Shapiro-Wilk test for normality (W-statistic = 0.9683) cannot reject the null hypothesis that GDDs are normally distributed (z = -1.121, p = 0.869).

3 METHODS

3.1 Model Specification

A key step in pricing weather derivative contracts is to estimate the expected value of the underlying weather index. We examine three methods: (i) historical burn analysis, (ii) an estimated econometric model, and (iii) a mean-reverting stochastic process with different parameters using Monte Carlo simulation. In the burn analysis, the average value over the previous decade is set as the estimated expected value for the contract year. Ten years are considered to be a reasonable time window for temperature data (Jewson et al. 2005). The mathematical formula is as follows:

$$[1] \qquad g_i = \sum_{k=i-10}^{i-1} g_k \,,$$

where g_i refers to growing degree days for year i, and g_k is growing degree days for years beginning a decade before year i.

As is shown in Figure 1, daily average temperatures clearly follow a sine function. Daily average temperatures are used to calculate GDDs, and an estimated sine function can be used to estimate daily average temperatures. The following functional form is assumed:

$$[2] \qquad \overline{T}_t = \sin(\omega t + \theta),$$

where $\overline{T}_t = ½ (T_t^{max} + T_t^{min})$ is the mean of the daily average temperature at day t (= 1, 2, …, 365 or 366). Thus, while average daily temperatures $\overline{T}_t$ follow a sine curve, the realized average temperature (T_t) on a given day t fluctuates randomly about that average. Further $\omega = 2\pi/365$ since the oscillation period is one year. As the yearly minimum and maximum mean temperatures do not usually occur at the troughs and peaks in Figure 1, a phase angle θ is introduced in the mean temperature model. In addition, as the global temperature gets warmer, there might be a positive upward trend in the data. Therefore, the model for the mean daily average temperature might have the following form:

[3] $$\overline{T}_t = b_0 + b_1 t + b_2 \sin(\omega t + \theta),$$

where b_i and θ are parameters to be estimated and t is a trend variable causing $\overline{T}_t$ to rise over time. We can then rewrite equation [3] as:

[4] $$\overline{T}_t = b_0 + b_1 t + b_2 (\cos \theta \times \sin \omega t + \sin \theta \times \cos \omega t)$$

$$= b_0 + b_1 t + a_2 \sin \omega t + a_3 \cos \omega t,$$

where b_0, b_1, a_2 (= $b_2 \cos \theta$) and a_3 (= $b_2 \sin \theta$) are parameters to be estimated.

As temperatures cannot rise or fall indefinitely, a stochastic process model cannot allow temperature to deviate much from its mean value in the long run. In other words, the stochastic process describing the temperature should have a mean-reverting property. Temperature can be modelled by the following mean-reverting process, which is an example of an Ito Process (Dixit and Pindyk 1994):

[5] $$dT_t = \alpha (\overline{T}_t - T_t) dt + \sigma_t dw_t$$

where $\alpha(\overline{T}_t - T_t)$ is a drift term and $\sigma_t dw_t$ is the dispersion of the Weiner process w_t (Brownian motion), with $dw_t \sim N(0 \sqrt{dt})$ and σ_t is the volatility of the daily average temperature. In this case, T_t is the realized or actual daily average temperature, $\bar{T}_t$ is the mean average temperature for day t, and α is the speed of reversion to the mean temperature. Thus, the stochastic difference equation [5] describes an Ornstein-Uhlenbeck process.

Because the drift term in [5] only ensures that temperatures revert toward the mean cyclical temperature (Figure 1), it is necessary to add a component that ensures the temperature also reverts toward the long-run average temperature. To do so, Alaton et al. (2002) added the following $d\bar{T}_t/dt$ term to the drift component:

[6] $$\frac{d\bar{T}_t}{dt} = a_1 + b_2\, \omega \cos(\omega t + \theta).$$

Then the mean-reverting process in equation [5] can be written as

[7] $$dT_t = [\alpha\,(\bar{T}_t - T_t) + \frac{d\bar{T}_t}{dt}]\, dt + \sigma_t\, dw_t.$$

Assuming the first day is s and the final day is t, the general solution to equation [7] is:

[8] $$T_t = \bar{T}_t + (T_s - \bar{T}_s)\, e^{-\alpha(t-s)} + \int_s^t e^{\alpha(t-\tau)} \sigma_\tau dw_\tau\ ,$$

where $\tau \in [s, t]$ and other terms are defined as previously.

3.2 Parameter Estimation

Any Δw corresponding to a time interval Δt satisfies the following equation (Dixit and Pindyk 1994; Alaton et al. 2002):

[9] $$\Delta w = \gamma_t\, (\Delta t)^{½}\ ,$$

where γ_t~N(0,1) is a random variable that is serially uncorrelated so E[γ_t, γ_s]=0 for $t \neq s$. As

Δt becomes infinitesimally small, we can represent the increment of a continuous Wiener process, dw, in time t as:

[10] $dw = \gamma_t \, (dt)^{1/2}$.

The temperature variation $\hat{\sigma}_t$ can be defined as (Alaton et al. 2002):

[11] $\hat{\sigma}_t = \frac{1}{N_{i,m}} \sum_{t=1}^{N_i} (T_t - T_{t-1})^2$,

where T_t is defined as above, and i is the number of periods (years) used to determine the average temperature $\overline{T}$; and $N_{i,m}$ is the number of days in month m in year i.

The speed at which the process reverts back to the mean (α) is an important parameter. Three methods are used to estimate the parameter: a first-order autoregressive process AR (1), a discrete-time data equation and a martingale estimation function. Consider first the AR(1) process for temperature:

[12] $T_t = c_0 + c_1 T_{t-1} + \delta_t$,

where c_0 and c_1 are parameters to be estimated, and δ_t is a normally distributed random variable with zero mean. The estimated parameter c_1, which measures the speed that today's temperature reverts back to yesterday's temperature, is identically the mean-reverting parameter α, so $\hat{\alpha}_1 = \hat{c}_1$.

The parameters of the mean-reverting process could also be estimated using the discrete-time data equation (Dixit and Pindyk 1994):

[13] $T_t - T_{t-1} = d_0 + d_1 T_{t-1} + \zeta_t$,

where d_0 and d_1 are parameters to be estimated, and ζ_t is a normally distributed random variable with zero mean. Then, by estimating the parameters in [13], we obtain a second estimate of the mean-reversion parameter as $\hat{\alpha}_2 = -\ln(1+\hat{d}_1)$.

Finally, the martingale estimation function can also be used to estimate α. Based on Bibby and Sørensen (1995), Alaton et al. (2002) derive the following estimate of the mean-reversion parameter:

$$[14] \quad \hat{\alpha}_3 = -\ln\left[\frac{\sum_{t=1}^{N}\frac{\left(\overline{T}_{t-1}-T_{t-1}\right)\left(T_t-\overline{T}_t\right)}{\sigma_{t-1}^2}}{\sum_{t=1}^{N}\frac{\left(\overline{T}_{t-1}-T_{t-1}\right)\left(T_{t-1}-\overline{T}_{t-1}\right)}{\sigma_{t-1}^2}}\right],$$

where $\overline{T}_t$ is the average daily temperature from the previously estimated sine-function, T_t is again the realized average temperature, and σ_t is the standard deviation of the realized daily average temperatures for day t.

3.3 Payoffs and Premiums of Put and Call Options

Farmers can purchase a put option in the event that the weather index (growing degree days) is too low, or a call option in the event that it is too high. From the standpoint of the buyers, the payoff functions for put and call contracts are given by (Jewson et al. 2005):

$$[15] \quad p(x)_{put} = \begin{cases} D(K_1 - x), & x \le K_1 \\ 0, & x > K_1 \end{cases},$$

$$[16] \quad p(x)_{call} = \begin{cases} 0, & x < K_2 \\ D(x - K_2), & x \ge K_2 \end{cases},$$

where $p(x)$ is the payoff; D is the tick size (dollar value per unit of the weather index); K_1 and K_2 are the strike (trigger) values for the put and call options, respectively; and x is the weather index. For put and call contracts, these are the payoffs against low and high values of the weather index, respectively.

Using historic daily average temperatures and assuming that the weather index employed for a financial instrument follows a normal distribution, the expected payoff is (Jewson et al. 2005):

[17] $$E_p = \int_{-\infty}^{\infty} f(x)p(x)dx,$$

where $f(x)$ is the probability density function (PDF) of rainfall, growing degree days or whatever measure is used for the weather index, and $p(x)$ is the payoff associated with the financial instrument for each outcome x of the weather variable or index. Denote the payoffs for put and call options as $p(x)_{put}$ and $p(x)_{call}$, respectively. Upon transforming the weather index into a standard normal distribution, the payoff function becomes,

[18] $$E_p = \frac{1}{\sigma}\int_{-\infty}^{\infty} \phi(z)p(x)dx,$$

where σ is the standard deviation of the weather index and $\phi(z)$ is the probability density function (PDF) of a standard normal distribution, $z = \frac{x-\mu}{\sigma}$ and $f(x) = \frac{\phi(z)}{\sigma}$.

Inserting payoff functions [15] and [16] for the put and call contracts into [18] gives the following respective closed-form functions for uncapped put and call options:

[19] $$E_{p,PUT} = \frac{1}{\sigma}\int_{-\infty}^{K_1} D(K_1 - x)\phi\left(\frac{x-\mu}{\sigma}\right)dx = D\sigma\phi\left(\frac{K_1-\mu}{\sigma}\right) + D\Phi\left(\frac{K_1-\mu}{\sigma}\right)(K_1-\mu),$$

[20] $$E_{p,CALL} = \frac{1}{\sigma}\int_{-\infty}^{K_1} D(x - K_2)\phi\left(\frac{x-\mu}{\sigma}\right)dx = D\sigma\phi\left(\frac{K_2-\mu}{\sigma}\right) + D(\mu-K_2)\left[1-\Phi\left(\frac{K_2-\mu}{\sigma}\right)\right],$$

where μ is the mean value of the weather index; K_1 and K_2 are the lower and upper strike values, respectively; ϕ and Φ refer to the normal probability density function and the cumulative probability distribution (CDF), respectively; and x is the weather index.

Let $k_1 = \dfrac{K_1 - \mu}{\sigma} = -m$ and $k_2 = \dfrac{K_2 - \mu}{\sigma} = m$, where $m = \{0.2, 0.4, \ldots, 2.0\}$. Then equations [19] and [20] can be written as:

[21] $E_{p,PUT} = D\, \sigma\, [\, \phi(-m) - m\, \Phi(-m)]$ and

[22] $E_{p,CALL} = D\, \sigma\, [\, \phi(m) - m + m\, \Phi(-m)]$.

The price of an option (or its premium) is calculated from the expected payoff as (Alaton et al. 2002):

[23] $c = e^{-r(u-v)} E_p$,

where c is the premium that the hedgers (buyers) need to pay for a contract, r is a risk-free periodic market interest rate, v is the date that the contract is issued (purchased), and u is the date the contract is claimed or the expiration date. For the stochastic model, E_p is based on predicted temperatures; for the weather index distribution model (discussed further below), it is based on the historic mean value of the corresponding weather index and its historic distribution. The seller of the option would expect a reward for taking on risk and, hence, the premium would be higher than the expected payoff by an amount known as the risk loading, which is often between 20% and 30% of the payoffs (Jewson et al. 2005). In the current application, we set the risk loading at 20% of the expected payoff of the contract.

4. RESULTS

The parameters of the sine-function are estimated using linear estimation and are provided in Table 1. Using the results from Table 1, we can write the equation for mean temperatures as:

[24] $\overline{T}_t = 8.18 + 15.8 \sin[(2\pi/365)\, t - 1.76]$.

From [24], it is evident that the average temperature is 8.18 ^{0}C, and there appears to be no warming trend. As the estimated parameter for the time-trend variable t is not significant, the model is re-estimated with the time trend variable removed. We can see that the magnitude of the remaining parameters changes only slightly, while $\overline{R}^2$ does not change. This model explains 89.4% of the variation in daily average temperatures. As shown in Figure 2, as the errors only deviate slightly from a normal distribution, it is assumed that the errors are normally distributed.

Table 1: Estimated Parameters for Sine Function, With and Without a Time Trend

	Estimated with time trend t			Estimated without time trend t		
Explanatory variable	Estimated Coefficient	Standard Error	prob> t-statistic	Estimated Coefficient	Standard Error	prob> t-statistic
t	0.000676	0.000922	0.463			
$\sin(\omega t)$	-2.92	0.137	0	-3.00	0.086	0
$\cos(\omega t)$	-15.52	0.086	0	-15.51	0.086	0
constant	8.05	0.179	0	8.18	0.061	0
$\overline{R}^2$	0.894			0.894		

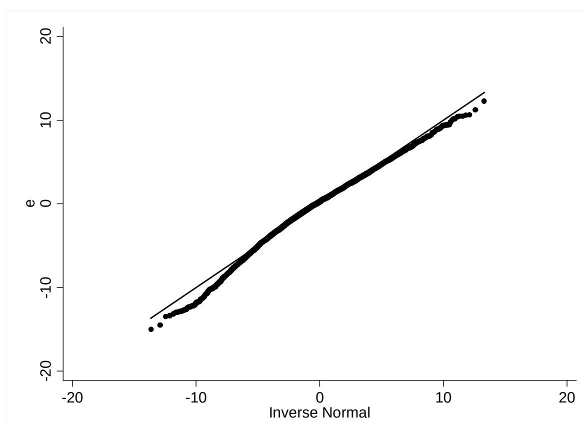

Figure 2: Quintiles of normal distribution plot for errors

From the 11-year historic daily temperature data, the estimated values of σ are given in Table 2. The regression results for two of the three alternative estimates for the mean-reverting speed parameter, namely, equations [12] and [13], are as follows:

$$[25] \quad T_t = \underset{(0.05)}{0.38} + \underset{(0.0033)}{0.9763}T_{t-1}, \ \bar{R}^2=0.956,$$

$$[26] \quad T_t - T_{t-1} = \underset{(0.0478)}{0.3801} - \underset{(0.0033)}{0.0237}T_{t-1}, \ \bar{R}^2=0.012,$$

The estimated values for the mean reversion speed are $\hat{\alpha}_1$ = 0.9763 (estimated from the AR (1) process) and $\hat{\alpha}_2$= –ln (1-0.0237) = 0.02399 (estimated from the discrete-time data equation). Finally, using martingale estimation function [14], we find $\hat{\alpha}_3$ =0.2698.

Table 2: Estimated σ for each of the 12 Months

Month	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
σ	2.72	2.68	3.43	3.45	2.88	2.28	2.27	1.95	2.28	2.42	2.45	2.79

Fluctuations of historic and estimated (predicted) daily average temperatures are provided in Figure 3, and these indicated that estimates using the sine function (solid line) fit the trend of the actual daily average temperatures (dots) quite closely. By adding a Wiener process to the sine function, we then simulate the daily average temperatures using Monte Carlo simulation with different mean reversion speeds – parameters from the AR (1) process, discrete-time data equation and the Martingale estimation function. We also predict the daily average temperatures only by the sine function without the stochastic process. Finally, we generate the weather index (GDDs) from the estimated daily average temperatures.

To compare the estimated growing degree days by different methods and with different mean reversion parameters, the variations between estimated and actual GDDs over the period of 2001 and 2011 are plotted in Figure 4, with values presented in Table 3. The absolute variations between estimated and actual GDDs, measured from the smallest to the largest, are those based on the historical average method (*H*), the stochastic process with a high mean reversion speed of 0.9763 (R_1), and the method based on the sine function without a stochastic process (*M*). The remaining estimated variations, R_2 (with mean reversion speed of 0.270) and R_3 (with mean reversion speed of 0.024), are much larger and, thus, are excluded from further analysis. In other words, the historical average values of the historical GDDs fit the actual GDDs best, followed by the simulated GDDs from the stochastic process with a high mean reversion speed (α_1) and by the sine function without a stochastic process.

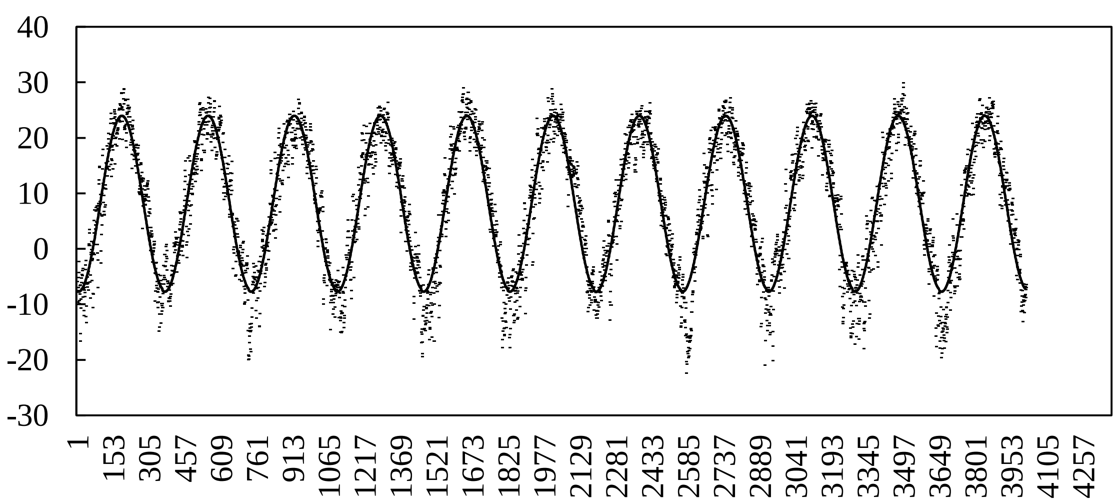

Figure 3: Fluctuation of historical daily average temperatures from estimated sine curve (line: estimated sine curve; dots: historical data)

In the mean reversion method, the performance in predicting growing degree days declines in going from the stochastic process with a high mean reversion speed (estimated from the autoregressive function) to the estimated sine function without a stochastic process. This is followed by the stochastic process with a low mean reversion speed (estimated from the martingale estimation function), with the stochastic process with a low mean reversion speed (estimated from the discrete-time data equation) proving to perform worst according to our criterion – the value of the mean value of the annual absolute deviations. Therefore, when pricing weather derivatives, the first choice is the weather index based on the distribution of historic means, followed by a method based on the mean reversion method with a high mean reversion parameter estimated from the autoregressive function.

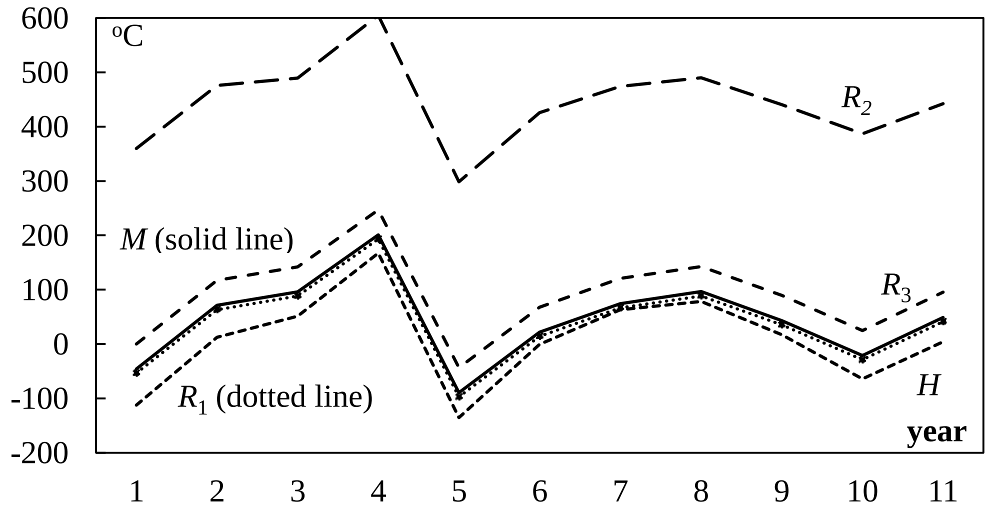

Figure 4: Differences between estimated GDDs and actual GDDs, 2001-2011 (Variations: M from sine-function; R_1 from stochastic model with $\hat{\alpha}_1$; R_2 from stochastic model with $\hat{\alpha}_2$; R_3 from stochastic model with $\hat{\alpha}_3$; H from average value for past decade)

Table 3: Variations between estimated and realized GDDs, 2001-2011[a]

Variation	Average of annual absolute variations[a]	Estimated expected value for GDD in 2012
Method 1: Sine function (M)	73.59	1494.91
Method 2: Mean reversion		
R_1	70.02	1487.06
R_2	444.24	1894.98
R_3	99.10	1540.98
Method 3: Historic average (H)	64.18	1455.71

[a] Used as the standard deviation of the GDD in the pricing of weather derivative contracts.

To price the financial weather derivatives, we assume a tick size D=\$1 and risk free interest rate r=0.08, Δt=3/4year (time between the issue date and the expiry date), and risk loading b=20%. Results for our study region in northern China are provided in Table 4. The premiums are the same for the estimated GDDs from the sine function and from the stochastic process with a high mean reversion speed; these, in turn, are 9% below those when GDDs are estimated from the historical average. As the GDDs from the historical average method track realized GDDs more closely, however, the premium from this method might well be more accurate. The method of the stochastic process with Monte Carlo simulation, or the econometric method using sine function, undervalues the premiums of the weather derivative contracts.

Table 4: Specification of GDD options[a]

Items	Put Option	Call Option
Weather Index	GDD	GDD
Strike Level (K_1 or K_2)	1455.71^0C–0.2×64.18^0C 1487.06^0C–0.2×70.02^0C 1494.91^0C –0.2×73.59^0C	1455.71^0C–0.2×64.18^0C 1487.06^0C –0.2×70.02^0C 1494.91^0C–0.2×73.59^0C
Tick Size (D)	$ 1	$ 1
Premium[b]	$ 21.27 $ 23.20 $ 24.39	$ 21.27 $ 23.20 $ 24.39
Payoff	Max (K_1 –GDD, 0)	Max (GDD–K_2, 0)
Issue date	December 31, 2011	December 31, 2011
Maturity date	September 31, 2012	September 31, 2012

[a] The strike values are μ–0.2σ, or m=0.2; when m=0.2, ϕ(-0.2)=0.3910, Φ(-0.2)=0.4207, ϕ(0.2)=0.3910, and Φ(0.2)=0.5792. The premiums are calculated from payoff equations [21] and [22], and using [23] plus a 20% risk loading factor to calculate the premiums.
[b] Premium for a standard deviation difference in mean weather index for the M, R_1 and H approaches to estimating GDDs.

5. CONCLUSIONS

The agricultural sector is particularly vulnerable to weather risks, but financial weather derivatives can be developed to reduce farmers' exposure to such risk. This may particularly be the case for developing counties where a large portion of the population is still dependent on agriculture and government insurance and other support is lagging. Indeed, studies have shown that farmers in central and northwestern China, for example, are interested in weather indexed insurance (Turvey et al. 2009; Liu et al. 2010). Given that farmers are interested in financial weather products in China, in this study we focused on the setting of premiums for puts and calls on growing-degree-day weather options. We used existing relationships between corn yields and weather parameters for northern China (Sun and van Kooten 2013).

We considered several models for forecasting future temperatures upon which to base a GDD weather index. These in turn would determine the premiums that markets would charge, excluding transaction costs. We investigated a more traditional burn analysis, which employed a simple historic temperature trend regression, several models that used a sophisticated stochastic process, and a Martingale approach. We found that a simple autoregressive AR(1) process led to the best approximation of realized temperatures and that premiums for options based on a GDD weather index derived from the estimated AR(1) model were lower than premiums derived from other methods. Further, if temperature was assumed to follow a stochastic process, the mean reversion parameter obtained from the AR(1) method gave a better result compared with other methods for mean reversion speed estimation.

Projecting future temperatures and growing degree days is fraught with uncertainty, which is why farmers wish to hedge against weather risk. However, markets need to provide farmers with hedges that are attractive, effective and truly representative of the risks producers encounter. Further research is required to better link crop yields to growing degree days – to match crop losses due to weather risks to the weather index – and to identify a proper tick size for pricing GDD-based weather derivatives.

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